One Postulate

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Abstract

Einstein built special relativity on two postulates. No experiment can distinguish one constant-velocity laboratory from another, and light travels at the same speed in every such laboratory. The first is a principle of symmetry. The second is a fact about a particular phenomenon, an observation imported into the foundations. We show this was unnecessary. The symmetry principle alone yields a family of possible universes, labelled by a single number κ. The Killing form (a diagnostic built into every set of symmetry rules, introduced by Wilhelm Killing in 1888) sorts them into exactly three kinds, one with no causality, one that cannot unify space and time, and one that achieves both and demands a finite speed that is the same in all frames. Experiment is needed to measure that speed, not to establish its existence. Einstein needed one postulate, not two.

The postulate

Einstein built special relativity on two principles.[1] The first is a symmetry requirement.

POSTULATE (relativity principle, 1905). *The laws of physics take the same form in all inertial frames.*

An inertial frame is any laboratory moving at constant velocity, a sealed room drifting through space. The postulate says that no experiment performed inside such a room can reveal how fast it is moving, or whether it is moving at all.

For Einstein this was more than a statement about experiments. It was a design principle. The mathematical framework of physics should contain no structure put in by hand. Everything should emerge from the rules themselves. Special relativity came from removing the assumption that all observers agree on which events are simultaneous. General relativity came from removing the assumption that there is a single natural way to label points in spacetime, the four-dimensional union of space and time. Each advance stripped away a background assumption that the previous framework had smuggled in.

The second postulate is different in character. It says that light travels at a finite speed c that is the same in all inertial frames. This is not a principle of symmetry. It is an observation about a particular physical phenomenon, imported into the foundations. For Einstein, who held that the laws of physics should be self-contained, this was a concession, an empirical fact doing the work of a structural argument.

What the postulate determines

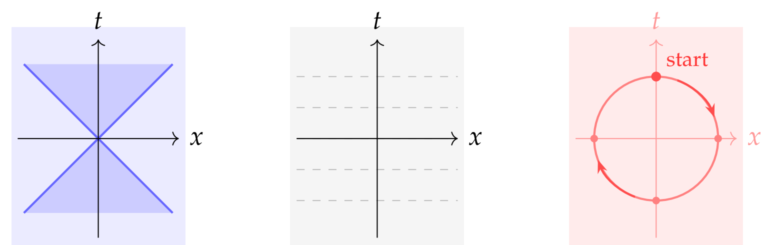
How do measurements in one inertial frame relate to those in another? If you are on a train moving at speed v relative to the platform, and you measure a time t and a position x, what are the platform's values t' and x'?

The relativity principle (that inertial-frame transformations are homogeneous, isotropic, and compose consistently) turns out to be extraordinarily restrictive. Since 1910, independent derivations[3,5–7] have shown that these properties allow exactly one family of transformation rules, governed by a single undetermined number κ.

$$t' = \gamma\,(t - \kappa\, v\, x), \qquad x' = \gamma\,(x - v\, t), \qquad \gamma = \frac{1}{\sqrt{1 - \kappa\, v^2}}.$$

These deserve a moment's reading. The second equation is intuitive, the platform sees your position shifted by the train's motion. The first is where the physics lives. The term $\kappa\, v\, x$ mixes space into time. It means that what counts as "now" depends on where you are, and κ controls the strength of this mixing. As v approaches $1/\sqrt{\kappa}$ (when $\kappa > 0$), the factor γ grows without bound, which acts as a speed limit, a velocity that can be approached but never reached. When $\kappa = 0$, the mixing vanishes. $t' = t$, time is absolute, and we recover Newton.

Three kinds of universe arise, depending on the sign of κ.



Lorentzian ($\kappa > 0$) $\mathfrak{so}(3{,}1)$ **Galilean** ($\kappa = 0$) $\mathfrak{iso}(3)$ **Euclidean** ($\kappa < 0$) $\mathfrak{so}(4)$

Lorentzian ($\kappa > 0$). The transformations are Einstein's. There is a finite maximum speed, and spacetime has lightcones, the V-shaped boundaries shown above, separating events that can influence each other from events that cannot. This is causal structure.

Galilean ($\kappa = 0$). The transformations are Newton's. Time is universal (the horizontal slices above, where every observer agrees on "now"), there is no speed limit, and space and time are independent of each other.

Euclidean ($\kappa < 0$). All four dimensions are equivalent. There is no lightcone, no distinction between past and future. All directions, including time, are geometrically the same. Change your velocity far enough and you loop back to rest.

Every treatment since 1910 has agreed that the first postulate alone can't determine the sign of κ.[3,4,6–11] To choose among the three, one must look outside, measure the speed of light, and thereby import Einstein's second postulate or something equivalent. Pauli wrote in 1921, "Nothing can naturally be said about the sign, magnitude and physical meaning of [κ]."

We disagree. The symmetry rules already contain the answer.

Can the rules examine themselves?

The three universes share the same generators and combination rules. They differ only in the value of κ, which controls how strongly the elementary operations interact with each other. Can this internal structure, using only its own resources, distinguish the three cases without any appeal to experiment?

It can. Wilhelm Killing found a way to do this in 1888,[2] seventeen years before Einstein's paper. His construction, the Killing form, applies to any set of symmetry rules.

Every set of symmetry rules is built from elementary operations called generators. For the symmetry of motion, there are six. Three rotations (J_1, J_2, J_3, one for each spatial axis) and three boosts (K_1, K_2, K_3, one for each direction of motion). A rotation relates your laboratory to one pointing a different way. A boost relates it to one moving past you, the operation of changing velocity. The relativity postulate is, at bottom, a statement about the boosts.

The rules of the algebra (a Lie algebra, the structure that encodes continuous symmetries) specify what happens when you perform these operations in sequence, and in particular, when the order matters. Rotations don't commute. Rotate an aeroplane nose-up and then roll it right, and you get a different orientation than if you roll first and pitch second. This sensitivity to ordering is what gives three-dimensional space its rigid geometric structure. Whether the *boosts* share this property depends on κ.

The combination rules for the six generators are fixed by the cited derivations. Rotations combine as $[J_i, J_j] = \epsilon_{ijk} J_k$ (the familiar angular momentum algebra). The boosts transform as vectors under rotation, $[J_i, K_j] = \epsilon_{ijk} K_k$. And the boosts combine with each other as $[K_i, K_j] = -\kappa\, \epsilon_{ijk} J_k$, so a boost followed by a boost in a different direction produces a rotation, with strength proportional to κ. When $\kappa = 0$, boosts commute and this link to rotation is severed.

Each generator acts as a machine that reshuffles the others (the adjoint action). When two generators are combined in sequence, the result may mix rotations into boosts and boosts into rotations. The Killing form measures the total reshuffling that each pair produces, $B(X, Y) = \mathrm{tr}(\mathrm{ad}_X \circ \mathrm{ad}_Y)$, the trace of the combined reshuffling. It produces a single number, computed entirely from the combination rules. It is the algebra's characteristic bilinear form, not introduced from outside, not a choice, but the algebra examining itself. Every set of symmetry rules has exactly one Killing form, just as every object has exactly one centre of mass.

For the symmetry rules labelled by κ, the computation amounts to tracking how each generator reshuffles the other five and summing the result. Consider the boost K_1. From the brackets, ad_{K_1} sends

$$\begin{array}{rcll} J_1 & \to & 0 & \\ J_2 & \to & K_3 & ([K_1, J_2] = K_3) \\ J_3 & \to & -K_2 & ([K_1, J_3] = -K_2) \\ K_1 & \to & 0 & \\ K_2 & \to & -\kappa\, J_3 & ([K_1, K_2] = -\kappa\, J_3) \\ K_3 & \to & \kappa\, J_2 & ([K_1, K_3] = \kappa\, J_2) \end{array}$$

Four generators are reshuffled; two map to zero. Now apply ad_{K_1} a second time.

$$J_2 \to K_3 \to \kappa\, J_2, \qquad J_3 \to -K_2 \to \kappa\, J_3, \qquad K_2 \to -\kappa\, J_3 \to \kappa\, K_2, \qquad K_3 \to \kappa\, J_2 \to \kappa\, K_3.$$

Each of the four returns to κ times itself. The result is diagonal, with $\mathrm{ad}^2_{K_1} = \kappa$ on the active sector. The trace is $B(K_1, K_1) = 4\kappa$. The same computation for a rotation gives $B(J_1, J_1) = -4$ (the four active generators each return to -1 times themselves). The complete Killing form is

$$B \;=\; \mathrm{diag}(\underbrace{-4,\ -4,\ -4}_{\text{rotations}},\ \underbrace{4\kappa,\ 4\kappa,\ 4\kappa}_{\text{boosts}}).$$

The rotations always register -4. The boosts register 4κ, and their value depends entirely on κ.

So we have the self-test. When the Killing form returns a non-zero value for a generator, the algebra "sees" that generator and can build geometry from it (the form is non-degenerate on that subspace). When it returns zero, the algebra is blind in that direction and the form degenerates.

The sign carries further information. Rotations, whose Killing form value is negative (-4), generate periodic transformations. Rotate 360 and return to the start. They are compact. The sign of the Killing form classifies each generator as periodic (negative, compact) or non-periodic (positive, non-compact). This is not a convention; it is a property of the exponential map from the algebra to the group. A negative value means the reshuffling reverses direction at each step, like the alternating signs in a cosine series, so the generated transformation oscillates and returns to its starting point. A positive value means the reshuffling reinforces itself at each step, like the uniform signs of a hyperbolic cosine, so the generated transformation grows without bound. The value on the boosts (4κ) now sorts the three universes.

Three verdicts

$\kappa < 0$**. No causal structure.**

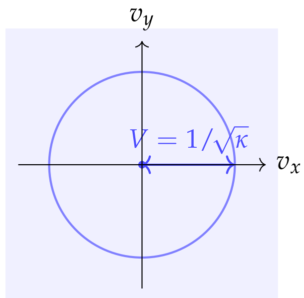
When κ is negative, the Killing form is non-zero on both rotations and boosts, so the algebra sees all of its own generators and has no blind spots. But the sign on the boosts is now negative ($4\kappa < 0$), the same sign as on the rotations ($-4 < 0$). Both are periodic. Both generate closed orbits. The algebra cannot distinguish which generators are rotations and which are boosts. Without that distinction, it cannot define velocity (displacement per unit time requires a non-periodic direction distinct from the periodic spatial rotations), and the boosts have no physical interpretation as changes of velocity.

$\kappa = 0$**. The algebra goes blind.**

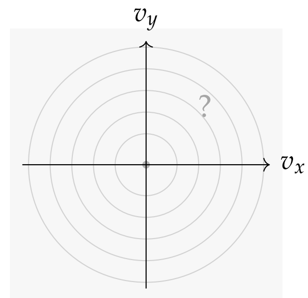
When $\kappa = 0$, the Killing form returns $4 \times 0 = 0$ on every boost generator. The self-test comes back blank for exactly the operations that the relativity postulate is about, the boosts that connect one inertial frame to another. The rotations still register their full -4. But the boosts are invisible to the algebra's own diagnostic.

The phrase "the same laws in all frames" can be read in two ways. The first is that the equations have the same *form*. $F = ma$ in every frame. The template is preserved. The second is that the equations have the same *content*, the same numerical predictions. These are different statements. At $\kappa = 0$, two observers can adopt different ratios of space units to time units and both satisfy every bracket relation. They agree that $F = ma$. They agree on the form of every equation. They disagree on what a second is worth relative to a metre, because the algebra has not fixed a conversion between the two. If one observer defines velocity as metres per second and another adopts a different ratio, they assign different kinetic energies to the same moving object, not because they disagree about physics, but because the framework has not told them how to relate their units. That is not "the same laws." It is the same template filled in differently. A framework whose predictions vary between observers is not a physical theory. The relativity postulate demands identical content, not just identical form. That requires a non-degenerate Killing form on the boost sector.

The blindness has a concrete consequence. Consider the space of all possible velocities. The boost generators span a three-dimensional subspace, and rotational symmetry forces this space to look the same in every direction, so its shape is fixed. (By Schur's lemma, the unique SO(3)-invariant form on the boost subspace is δ_{ij}, up to a positive constant.) But its *scale* is not. How large is a unit of velocity? Is there a natural ruler? The transformation equations already showed that γ diverges at $v = 1/\sqrt{\kappa}$, the invariant speed, the maximum velocity built into the geometry. The Killing form is what determines whether that speed is real. $B(K_i, K_j) = 4\kappa\, \delta_{ij}$, which is positive only when $\kappa > 0$. At $\kappa = 0$, the Killing form returns zero, the invariant speed is undefined, and no natural ruler exists. The shape of the room is known, but the ruler is missing.



$\kappa > 0$. Killing form sets the ruler



$\kappa = 0$. Shape fixed, scale lost

At $\kappa > 0$ (left), the Killing form fixes the radius of velocity space. There is a definite invariant speed. At $\kappa = 0$ (right), the shape is spherically symmetric (rotational symmetry guarantees that) but every circle is equally valid. The algebra can't choose among them. No invariant speed is determined.

This is not a matter of unit choice. Ordinary changes of units (metres to centimetres, say) rescale all spatial and temporal generators together, leaving their ratio unchanged. But the Galilean algebra allows something stronger. Suppose an observer rescales every spatial generator by the same factor α, sending $P_i \to \alpha P_i$ and $K_i \to \alpha K_i$, while leaving H and J_i unchanged. This changes the spatial ruler without touching the clock. At $\kappa = 0$, every bracket relation survives. $[K_i, H] = P_i$ becomes $[\alpha K_i, H] = \alpha P_i$, consistent; $[K_i, P_j] = 0$ becomes $[\alpha K_i, \alpha P_j] = 0$, consistent. All rotation brackets are unchanged. The rescaled generators satisfy exactly the same rules as the originals. Two observers who adopt different values of α agree on every equation and disagree on every measurement involving length. The algebra cannot tell them apart.

At $\kappa > 0$, the same rescaling fails. The bracket $[K_i, P_j] = \kappa\, \delta_{ij}\, H$ becomes $[\alpha K_i, \alpha P_j] = \alpha^2 \kappa\, \delta_{ij}\, H$. This matches the original only if $\alpha = 1$. The algebra locks the spatial scale to the temporal one through κ, and no independent rescaling is possible. That is what the invariant speed does. It welds space to time.

The same failure shows up in spacetime. When $\kappa = 0$, the time equation becomes simply $t' = t$. Time is untouched by the boosts. Every observer in every inertial frame agrees on what time it is. This is Newton's absolute time, a universal clock that ticks the same for everyone. The algebra preserves this structure but doesn't *generate* it. Absolute time is not an approximation, not what relativistic time looks like when things move slowly. It is a fixed point of the Galilean symmetry, something the symmetry leaves entirely alone, hardwired into the $\kappa = 0$ algebra. (The covector dt is invariant under the full homogeneous algebra.) It is a background structure, assumed rather than derived. The spatial geometry must also be supplied from outside.

This can be made algebraically precise. Add time translation H and spatial translations P_i to the algebra. The Jacobi identity (applied to triples such as K_i, K_j, H and using the already-established brackets) forces $[K_i, H] = P_i$ and $[K_i, P_j] = \kappa\, \delta_{ij}\, H$. Now look for the unique G-invariant bilinear form q on the translation module $V = \text{span}\{H, P_i\}$. Rotational symmetry requires $q(H, H) = A$, $q(H, P_i) = 0$, $q(P_i, P_j) = B\, \delta_{ij}$. Boost invariance forces $B = -A\kappa$. When $\kappa > 0$, $q = \text{diag}(A, -A\kappa, -A\kappa, -A\kappa)$. Full rank. The Minkowski metric, determined by the algebra. When $\kappa = 0$, $B = 0$. The form is $q = \text{diag}(A, 0, 0, 0)$. Rank one. The algebra provides a time metric and nothing else. The spatial Euclidean metric that Galilean physics requires is not supplied by the algebra. It must be added from outside.

Newton's framework requires absolute time and absolute space, supplied from outside, to produce definite answers. That external structure is what converts the template into a theory. It is also what the relativity postulate forbids. If "the same" requires supplementary definitions, the laws are not the same, only their form is.

Why does this happen? Because at $\kappa = 0$, the boosts commute ($[K_i, K_j] = 0$). Performing boost A followed by boost B gives the same result as B followed by A. Non-commutativity is what gives rotations their geometric rigidity. When the boosts commute, they lose that same rigidity. They can't knit space and time together. The mixing of space and time in Einstein's theory is a direct consequence of the fact that, at $\kappa > 0$, the order of boosts matters ($[K_i, K_j] = -\kappa\, \epsilon_{ijk} J_k$, linking boosts back to rotations).

All of these failures trace to a single root, $[K_i, K_j] = 0$. The degenerate Killing form, the missing velocity-space ruler, the decomposition of spacetime into separate time and space sectors, the spacetime metric collapsing to dt^2 alone. Galilean mechanics is the low-velocity limit of Lorentzian kinematics, not evidence for an independent framework. And the passage between the two isn't a smooth limit; it is a structural break (an Inönü-Wigner contraction, where the algebraic structure changes discontinuously). The Galilean algebra is a different kind of object from the Lorentzian one.

$\kappa > 0$**. Everything is determined.**

When κ is positive, the Killing form returns -4 on the rotations and 4κ on the boosts. Every generator is visible. No direction is blind.

Velocity space now has a definite geometry, and the ruler is fixed. The Killing form gives $B(K_i, K_j) = 4\kappa\, \delta_{ij}$, positive and non-degenerate, confirming that the boost sector is non-compact. The invariant speed $V = 1/\sqrt{\kappa}$ (at which γ diverges in the transformation equations) is finite, real, and the same in all frames. Unlike the Galilean case, the scale isn't left dangling. The symmetry rules set it.

Spacetime also acquires a definite geometry, and here the contrast with $\kappa = 0$ is sharpest. At $\kappa = 0$, every observer agrees on time intervals; duration is frame-independent. But at $\kappa > 0$, no observer's time is privileged. Neither time intervals nor spatial distances are frame-independent on their own. The only quantity all observers agree on is the spacetime interval, a single number built from both spatial and temporal separations that combines space and time inseparably. (The invariant metric is $g \propto \mathrm{diag}(1, -\kappa, -\kappa, -\kappa)$; with $\kappa = 1/c^2$, this is $ds^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2$. The algebra determines this quadratic form uniquely, by Schur's lemma on the irreducible four-dimensional representation. When a symmetry group acts on a space that cannot be decomposed into smaller independent pieces, any invariant quadratic form on that space is unique up to an overall scale factor.) That is what unification means. The only invariant quantity mixes both.

This geometry has lightcones. Past and future are distinguished. Causal structure exists.

The symmetry rules determine every structural feature, the invariant speed, the metric, the causal structure. Nothing is left free except the numerical scale. The postulate, taken at its word, yields exactly one kind of universe.

	$\kappa < 0$	$\kappa = 0$	$\kappa > 0$
Killing form on boosts	$4\kappa < 0$	0	$4\kappa > 0$
Invariant speed $V = 1/\sqrt{\kappa}$	imaginary	undefined	finite, real
Spacetime metric	Euclidean	dt^2 only	Lorentzian
Causal structure	none	none	lightcones
Space–time unification	all alike	impossible	complete
Background structure needed	none	yes (time, spatial metric)	none

Structure and scale

A finite invariant speed must exist, and spacetime must be Lorentzian. What the argument doesn't determine is the numerical value of that speed. V depends on κ, which sets the conversion factor between units of space and units of time. Measuring $c \approx 3 \times 10^8$ m/s fixes $\kappa = 1/c^2$. But this is calibration, fitting a number to a framework whose architecture is already determined.

Why does this matter? Because Einstein's second postulate was calibration dressed up as foundation. The symmetry rules themselves demand a finite invariant speed. Experiment tells us *how fast*. The algebra tells us *that*.

Einstein was right that the relativity principle was the deeper of his two postulates, and right that background structure should be eliminated. The tools to finish the job existed in his time. The computation requires nothing beyond the symmetry rules and some algebra. But the conclusion is the one he reached. There is a finite invariant speed, spacetime is Lorentzian, and space and time are unified.

The speed's value is empirical. Its existence is not.

Einstein needed one postulate, not two.

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