The Cosmological Constant Is Positive

Emad Mostaque
Intelligent Internet

April 2026

Abstract

Einstein introduced the cosmological constant Λ in 1917 to obtain a self-contained universe, one whose physics required no boundary conditions at infinity. He withdrew it when expansion was observed, reportedly calling it his greatest blunder. We show the sign of Λ was already determined by his own postulate. The relativity principle, read precisely, requires the algebra of spacetime symmetries to determine the physics completely, with nothing supplied from outside. Given the Lorentz algebra, the only consistent way to add translations produces a one-parameter family of algebras labelled by Λ. The eigenvalue spectrum of the algebra's built-in diagnostic (the Killing form, introduced by Wilhelm Killing in 1888) settles the sign. At $\Lambda = 0$ the algebra has a blind sector and cannot fix a length scale; at $\Lambda < 0$ the resulting spacetime requires boundary conditions that the postulate does not provide. Only at $\Lambda > 0$ does the algebra determine a Lorentzian metric and yield a spacetime whose physics is fully fixed by the postulate. That spacetime is de Sitter. The relativity principle does not merely permit de Sitter as one option among three; it singles out de Sitter as the unique admissible vacuum kinematics. The withdrawal of Λ was the blunder. The introduction was the closest Einstein came to deriving the spacetime his own postulate required.

The postulate

Einstein built general relativity on a single principle:[1,3] the framework must contain no external structure.

THE RELATIVITY PRINCIPLE (1905[1]). *The laws of physics are the same in all inertial frames and at all spacetime points.*

For Einstein this was a prohibition. No preferred frame, no preferred origin, no structure put in by hand. Algebraically, the postulate defines a Lie group G acting transitively on the space of inertial frames (every frame equivalent to every other, none preferred). The Lie algebra $\mathfrak{g}$ carries a characteristic bilinear form $B(X, Y) = \mathrm{Tr}(\mathrm{ad}_X \circ \mathrm{ad}_Y)$, the Killing form, where $\mathrm{ad}_X(Y) = [X, Y]$ sends each generator Y to its bracket with X. The precise reading of "the same laws," not just "the same form," requires B to be nondegenerate on every sector of the algebra (every subspace spanned by a subset of generators). The weak reading (same syntactic form of equations) is a condition on how laws are written, not on what they say. As Kretschmann showed in 1917,[7] any theory can be dressed up to have "the same form" in different frames by adjoining auxiliary structure (preferred-frame fields, background metrics, fixed connections) and writing equations covariantly with respect to that structure. Such theories satisfy the weak reading while violating its spirit, the very spirit Einstein invoked to reject Newtonian absolute space. The strong reading is therefore not a strengthening of the postulate. It is the minimum content at which the postulate distinguishes Einstein's program from the programs it was designed to replace. Algebraically, that minimum content is non-degeneracy of B. A sector where B vanishes is one the algebra cannot see from within, meaning any structure there is external scaffolding. Special relativity (1905) came from removing absolute simultaneity. General covariance (1915[3]) came from removing the preferred coordinate system. Each advance stripped away a background assumption that the previous framework had smuggled in.

In particular, if a theory requires external data to determine its physics (boundary conditions the framework doesn't generate, or a metric the algebra doesn't fix) the theory isn't self-contained and the postulate is violated. Einstein's equivalence principle (1907[2]), the demand that the metric emerge from the framework rather than be imposed, is a consequence of this reading. An externally supplied metric is external structure, which the postulate forbids.

What the postulate determines

The relativity principle requires Lorentz symmetry: the rules of special relativity that relate different inertial frames. Six generators (three rotations and three boosts) encode these rules and form the Lorentz algebra ($\mathfrak{so}(3,1)$).

But inertial frames can also be displaced in space and time. The laws of physics shouldn't depend on where or when you are. This adds four more generators, one for each direction of spacetime, called translations (P_0, P_1, P_2, P_3). The full algebra now has ten generators.

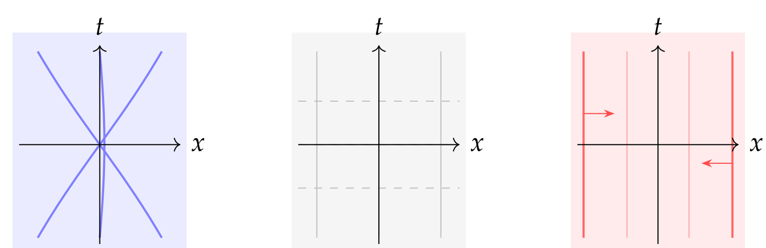
How do the translations interact with each other? The Lorentz part of the algebra is already fixed. The translations form a Lorentz 4-vector, so their transformation under the Lorentz generators is fixed ($[M_{\mu\nu}, P_\rho] = \eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu$, with signature $(+,-,-,-)$ throughout). What remains open is the bracket between translations, $[P_\mu, P_\nu]$.

The answer is tightly constrained. The bracket must be antisymmetric and must transform as a Lorentz tensor. The most general ansatz is $[P_\mu, P_\nu] = \alpha M_{\mu\nu} + c_{\mu\nu}{}^{\rho} P_\rho$: a piece proportional to the Lorentz generators and a piece mixing translations among themselves via some constant tensor $c_{\mu\nu}{}^{\rho}$. Under parity-preserving spatial isotropy, no such Lorentz-invariant tensor exists antisymmetric in $\mu\nu$ (the only candidate, the parity-odd $\epsilon_{\mu\nu\rho\sigma}P^{\sigma}$, is also incompatible with the Jacobi identity). The full kinematical-algebra classification[8,9] confirms this exhausts the possibilities. The algebra's rules must be internally consistent: when three generators interact, the result can't depend on the order in which you pair them. (This consistency condition is called the Jacobi identity.) Jacobi forces α to be a constant, leaving:

$$[P_\mu, P_\nu] = -\Lambda\, M_{\mu\nu},$$

with $\Lambda \in \mathbb{R}$, a single undetermined number, the cosmological constant.

Three kinds of algebra arise, depending on the sign of Λ.



de Sitter ($\Lambda > 0$) $\mathfrak{so}$(4,1) **Minkowski** ($\Lambda = 0$) $\mathfrak{iso}$(3,1) **Anti-de Sitter** ($\Lambda < 0$) $\mathfrak{so}$(3,2)

De Sitter ($\Lambda > 0$). Spacetime has positive curvature. The algebra is $\mathfrak{so}(4,1)$.

Minkowski ($\Lambda = 0$). Spacetime is flat. Translations commute ($[P_\mu, P_\nu] = 0$). The algebra is the Poincaré algebra $\mathfrak{iso}(3,1)$.

Anti-de Sitter ($\Lambda < 0$). Spacetime has negative curvature. The thick vertical lines in the diagram represent the boundary at spatial infinity, which is timelike: information can flow in from the edge, as the arrows indicate. The algebra is $\mathfrak{so}(3,2)$.

Can the algebra settle the sign?

This family was classified by Bacry and Lévy-Leblond[8] and Bacry and Nuyts.[9] Every previous treatment has concluded that the sign of Λ can't be determined without empirical input. We disagree. The algebra already contains the answer.

The Killing form, introduced by Wilhelm Killing in 1888, is the characteristic bilinear form of any Lie algebra. It is computed entirely from the structure constants, not introduced from outside, not a choice. On any simple Lie algebra it is the unique invariant bilinear form up to a positive scalar, and the only freedom is an overall normalization that doesn't affect signs or non-degeneracy. The algebras at $\Lambda \neq 0$ ($\mathfrak{so}(4,1)$ and $\mathfrak{so}(3,2)$) are simple (they have no nontrivial internal sub-structure that all brackets preserve), so on them this uniqueness is absolute. The criterion "non-degenerate on every sector" is therefore not a choice between diagnostics. It is the unique algebraic statement of "the framework registers every generator from within itself." To reject the Killing form is to reject any purely internal diagnostic, which is to abandon self-containment at the outset. Every algebra has exactly one Killing form.

Applied to the six-dimensional Lorentz algebra alone, the same diagnostic establishes that an invariant speed must exist and that spacetime must be Lorentzian.[15] We now apply it to the full ten-dimensional kinematical algebra.

For the ten-dimensional algebra labelled by Λ, we compute the Killing form on the time translation P_0. The relevant brackets are $[K_i, P_0] = P_i$ (a boost converts time translation into space translation) and $[P_0, P_i] = -\Lambda\, K_i$ (from $[P_\mu, P_\nu] = -\Lambda\, M_{\mu\nu}$ above). From these, ad_{P_0} sends:

$$\begin{array}{lcll} J_i & \to & 0 & ([P_0, J_i] = 0) \\ K_i & \to & -P_i & ([P_0, K_i] = -P_i) \\ P_0 & \to & 0 & \\ P_i & \to & -\Lambda\, K_i & ([P_0, P_i] = -\Lambda\, K_i) \end{array}$$

Six generators are reshuffled; four map to zero. Applying ad_{P_0} a second time:

$$K_i \to -P_i \to \Lambda\, K_i, \qquad P_i \to -\Lambda\, K_i \to \Lambda\, P_i.$$

Each of the six returns to Λ times itself. The result is diagonal: $\mathrm{ad}^2_{P_0} = \Lambda$ on the active sector. The trace is:

$$B(P_0, P_0) \;=\; 6\Lambda.$$

The Killing form is a one-number summary. The full diagnostic is the eigenvalue spectrum of $\mathrm{ad}^2_{P_0}$ on the rest of the algebra, and the Killing form is the trace of that spectrum. The spectrum carries strictly more structural information than its trace. It tells us about compactness, periodicity, and scale, not just about a single sign. The argument that follows reads the eigenvalues directly; the Killing form's sign follows as the trace.

The sign of the Killing form on time is the sign of Λ. The eigenvalues of $\mathrm{ad}^2_{P_0}$ decide everything. On the active sector, $\mathrm{ad}^2_{P_0} = \Lambda$, so ad_{P_0} has eigenvalues $\pm\sqrt{\Lambda}$. The one-parameter subgroup generated by P_0 is $\exp(t\,\mathrm{ad}_{P_0})$, a matrix whose entries are controlled by $e^{\pm t\sqrt{\Lambda}}$. The sign of Λ therefore determines the character of time evolution. When $\Lambda > 0$, the eigenvalues are positive, the exponential $\exp(t\sqrt{\Lambda})$ is real and non-periodic, and time extends indefinitely. When $\Lambda < 0$, the eigenvalues are negative, the exponential $\exp(it\sqrt{|\Lambda|})$ is periodic, and $\mathrm{SO}(3,2)$ has periodic time. Its universal cover does not. The algebra cannot determine which. When $\Lambda = 0$, the eigenvalues are zero, ad_{P_0} is nilpotent on translations, the generator has no characteristic scale, and the algebra is blind. The Killing form $B(P_0, P_0) = 6\Lambda$ is the trace, the sum of these eigenvalues. Its sign is theirs.

The sign classifies each generator. Negative means periodic (compact, like a rotation, which returns to its starting point), positive means non-periodic (non-compact, like a boost, which extends indefinitely), and zero means the algebra is blind. The spatial translations give $B(P_i, P_j) = -6\Lambda\,\delta_{ij}$ (opposite sign, just as the spatial rotations give the opposite sign to the boosts). The Lorentz generators always register a non-zero value. The translations register a value proportional to Λ. When $\Lambda = 0$, the Killing form goes blind on the entire translation sector.

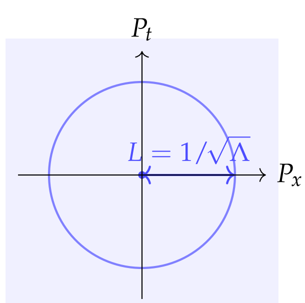
Three verdicts

$\Lambda = 0$: The algebra is not self-contained.

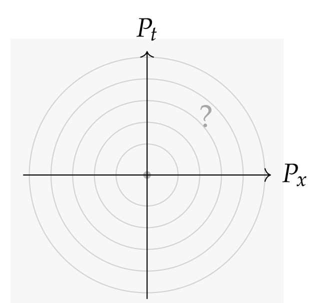
When $\Lambda = 0$, the translations commute ($[P_\mu, P_\nu] = 0$) and the Killing form returns zero on every translation generator. The distinction is structural, not quantitative: the algebra is semisimple (its Killing form is non-degenerate) for any $\Lambda \neq 0$, however small, and non-semisimple at $\Lambda = 0$. The structure type changes discontinuously (an Inönü–Wigner contraction[10]). Minkowski spacetime is the low-curvature limit of de Sitter spacetime, not evidence for a different algebra.

Lorentz symmetry fixes the *shape* of the spacetime metric, which must be proportional to $\eta_{\mu\nu}$. (By Schur's lemma, since the P_μ form an irreducible Lorentz 4-vector, $\eta_{\mu\nu}$ is the unique invariant form up to a positive constant.) But the *scale* is not fixed. The undetermined scale is the curvature radius, set by Λ itself. The algebra determines the metric only up to an overall positive constant, a shape without a scale, but not a unique metric.

This is not a minor ambiguity. The Poincaré algebra contains no length scale at all: ad_{P_0} is nilpotent on translations, $B(P_0, P_0) = 0$, and the structure constants are invariant under $P_\mu \to \lambda P_\mu$ for any $\lambda > 0$. This rescaling is an automorphism of the Poincaré algebra: it preserves every bracket, so the algebra literally cannot distinguish P_μ from λP_μ. The undetermined constant in the metric $c\,\eta_{\mu\nu}$ is not a physical parameter awaiting measurement; it is an algebraically meaningless artifact that the algebra's own symmetries can set to anything. By contrast, at $\Lambda \neq 0$ the same rescaling $P_\mu \to \lambda P_\mu$ sends $[P_\mu, P_\nu] = -\Lambda M_{\mu\nu}$ to $-\lambda^2 \Lambda M_{\mu\nu}$, which changes the structure constants unless $\lambda = 1$. The algebra is rigid: the bracket $[P_\mu, P_\nu] = -\Lambda M_{\mu\nu}$ locks the ratio between translations and Lorentz generators, and that locked ratio is the curvature scale. At $\Lambda = 0$, the lock opens, and the scale becomes a gauge freedom of the algebra rather than a physical observable within it.



$\Lambda > 0$: Killing form sets the scale



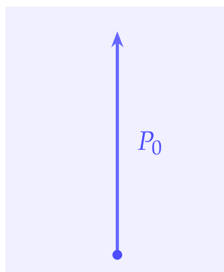
$\Lambda = 0$: shape fixed, scale lost

At $\Lambda > 0$ (left), the Killing form fixes the curvature radius: the metric has a definite scale. At $\Lambda = 0$ (right), every circle is equally valid. The algebra can't choose among them. A framework that requires an externally supplied scale is not self-contained. The postulate is violated.

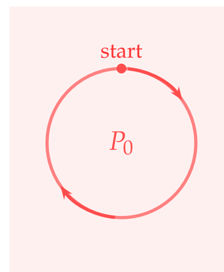
$\Lambda < 0$: The universe isn't self-contained.

When $\Lambda < 0$, the Killing form is non-zero on both the Lorentz generators and the translations, so the algebra has no blind spots. But the eigenvalues of $\mathrm{ad}^2_{P_0}$ are negative ($\Lambda < 0$). The exponential is $\exp(it\sqrt{|\Lambda|})$: periodic, with period $2\pi/\sqrt{|\Lambda|}$. The time translation generator P_0 is compact. The Killing form confirms this: $B(P_0, P_0) = 6\Lambda < 0$, the same sign as the rotations. The algebra classifies time translation as structurally the same as rotation. It treats "later" and "earlier" the way a rotation treats "clockwise" and "anticlockwise."

Compact P_0 erases the distinction between translation and rotation. The consequence is a physical ambiguity. The algebra $\mathfrak{so}(3,2)$ can be realised by different groups, one in which time is genuinely periodic (closed timelike curves), and one (its universal cover) in which it is not. The algebra does not determine which. Different groups, different physics, same rules. Compare with $\Lambda > 0$: the eigenvalues are positive, the exponential $\exp(t\sqrt{\Lambda})$ is real and non-periodic, no discrete quotient makes a real exponential periodic. The periodic-time pathology does not arise.



$\Lambda > 0$: P_0 is non-compact (extends indefinitely)



$\Lambda < 0$: P_0 is compact (loops back)

Independently, the spacetime of $\mathfrak{so}(3,2)$ (anti-de Sitter space) is not globally hyperbolic.[12] In a globally hyperbolic spacetime, specifying the state of the universe on a single spatial slice determines everything, past and future, with no further input. Anti-de Sitter space doesn't have this property. Its boundary at spatial infinity is timelike (the thick vertical lines in the three-panel spacetime diagram), which means information can flow in from the edge of spacetime. Determining the physics requires not just initial conditions but also boundary conditions at infinity.[11] Different boundary conditions give different physics from the same algebra.

The theory isn't self-contained, which the postulate forbids. This is the situation Einstein sought to avoid when he introduced Λ in 1917.[4]

$\Lambda > 0$: The postulate is satisfied.

When Λ is positive, the algebra $\mathfrak{so}(4,1)$ is the symmetry of a five-dimensional space with signature $(4,1)$, four spacelike and one timelike dimension. De Sitter spacetime lives as a four-dimensional surface inside this five-dimensional space; the extra dimension is spacelike, so no new timelike direction is introduced. The time translation generator P_0 is a boost (non-compact, $B(P_0, P_0) > 0$), matching the Lorentz boosts. Spatial translations are periodic ($B(P_i, P_i) < 0$), matching spatial rotations. This reflects the fact that de Sitter spatial sections are three-spheres (S^3), so a spatial translation that goes far enough eventually returns to its starting point. The algebra is semisimple with a non-degenerate Killing form.

The Killing form determines the spacetime metric, fixing both the Lorentzian signature (time and space distinguished by sign) and the scale (the curvature radius $L = 1/\sqrt{\Lambda}$). The metric emerges from the algebra; it is not assumed.

De Sitter spacetime is globally hyperbolic. Specifying the state of the universe on a single spatial slice determines the field evolution everywhere, with no boundary conditions needed. The postulate is satisfied. The physics is fully determined by the framework.

	$\Lambda < 0$	$\Lambda = 0$	$\Lambda > 0$
Eigenvalues of $\mathrm{ad}^2_{P_0}$ on translations	Λ (< 0)	0 (nilpotent)	Λ (> 0)
Time-translation P_0 character	compact	no scale	non-compact
Lie group(s) from algebra	ambiguous	unique	unique
Killing form $B(P_0, P_0)$ (trace)	< 0	0	> 0
Algebra	$\mathfrak{so}(3,2)$	$\mathfrak{iso}(3,1)$	$\mathfrak{so}(4,1)$
Spacetime G/H	anti-de Sitter	Minkowski (no scale)	de Sitter
Curvature radius	$1/\sqrt{\lvert\Lambda\rvert}$	undefined	$1/\sqrt{\Lambda}$
Metric determined by algebra?	no	no	yes
Globally hyperbolic?	no	yes	yes
Boundary conditions needed?	yes	no	no
Postulate (self-contained)?	**violated**	**violated**	**satisfied**

The unique admissible vacuum

The algebra $\mathfrak{so}(4,1)$ does not merely classify the symmetries of a hypothetical spacetime. It constructs one. The idea is standard: for any kinematical algebra with a Lorentz subalgebra, there is a natural spacetime whose points are the positions the symmetry group can reach from a single origin, with the Lorentz subgroup $H = SO(3,1)$ identified as the transformations that leave that origin fixed. The resulting space of points is the quotient G/H, where G is the full symmetry group. The metric on this space comes from the Killing form restricted to the translation directions (the generators not in the Lorentz subalgebra). For $G = SO(4,1)$, this construction yields four-dimensional de Sitter spacetime, with curvature radius $L = 1/\sqrt{\Lambda}$, naturally extended to its maximal global form $\mathbb{R} \times S^3$.

Three results that look independent are one. *Which sign of* Λ*?* Positive. *Which algebra?* $\mathfrak{so}(4,1)$. *Which spacetime?* De Sitter. The eigenvalues of $\mathrm{ad}^2_{P_0}$ acting on translations fix all three at once: their sign is the sign of Λ, the algebra they generate is $\mathfrak{so}(4,1)$, and the homogeneous-space construction on that algebra is de Sitter. There is no separate empirical question "is the vacuum de Sitter?" Once the algebra is fixed, the vacuum is fixed.

Structure and scale

Λ must be positive and the vacuum spacetime must be de Sitter. The argument fixes the sign, not the numerical value. The value sets the curvature radius of the vacuum, $L = 1/\sqrt{\Lambda}$. Measuring the accelerated expansion of the universe[13,14] calibrates the magnitude. But this is calibration, fitting a number to a framework whose architecture is already determined. The much-discussed cosmological constant *problem*, why Λ is small, is a separate question this paper does not address. Minkowski spacetime remains an excellent local approximation. It is the tangent-space limit of de Sitter at $L \approx 10^{26}$ m, vast compared to any laboratory or galactic scale.

The effort to measure Λ has become one of the largest enterprises in observational science. The Dark Energy Survey mapped hundreds of millions of galaxies. The Dark Energy Spectroscopic Instrument is building the largest three-dimensional map of the universe ever constructed. The Euclid space telescope, launched in 2023, is surveying a third of the sky. Billions of dollars in telescopes, trained on a single constant.

No first-principles derivation has determined its sign. The argument of this paper is that it was derivable from the start.

Since $\Lambda > 0$ forces the vacuum to be de Sitter, and de Sitter admits expanding cosmological foliations (slicings of spacetime into spatial surfaces that grow over time), the postulate predicts expansion of the vacuum. This is, to our knowledge, the first derivation of a positive cosmological constant from first principles alone.

Every ingredient was available by 1915. The Killing form (1888), the Lorentz algebra (1905), and general relativity (1915). The derivation of $[P_\mu, P_\nu] = -\Lambda\, M_{\mu\nu}$ requires only Lorentz representation theory and the Jacobi identity, both available to Einstein.

1905–1915. Einstein's postulate $\rightarrow$ $\Lambda > 0$ (this paper).

1917. De Sitter[5] finds the $\Lambda > 0$ vacuum solution, exponential expansion.

1922. Friedmann[6] finds expanding solutions with matter.

1929. Hubble confirms expansion observationally.

1998. Riess *et al.*[13] and Perlmutter *et al.*[14] confirm *accelerated* expansion, $\Lambda > 0$.

The derivation was available eighty-three years before its specific confirmation.

Einstein introduced Λ in 1917[4] to prevent expansion. His own postulate requires a Λ that guarantees it. He withdrew the cosmological constant and reportedly called it his greatest blunder.

The blunder was the withdrawal.

In 1917, Einstein wrote the sentence that anticipates the conclusion of this paper.

> *"We finally infer that boundary conditions in spatial infinity fall away altogether, because the universal continuum in respect of its spatial dimensions is to be viewed as a self-contained continuum of finite spatial volume."*
>
> (A. Einstein, 1917[4])

References

[1] A. Einstein, "Zur Elektrodynamik bewegter Körper," *Ann. Phys.* **17**, 891 (1905).

[2] A. Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen," *Jahrb. Radioakt. Elektron.* **4**, 411 (1907).

[3] A. Einstein, "Die Feldgleichungen der Gravitation," *Sitzungsber. Preuss. Akad. Wiss.*, 844 (1915).

[4] A. Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," *Sitzungsber. Preuss. Akad. Wiss.*, 142 (1917).

[5] W. de Sitter, "On the relativity of inertia," *Proc. Kon. Ned. Akad. Wet.* **19**, 1217 (1917).

[6] A. Friedmann, "Über die Krümmung des Raumes," *Z. Phys.* **10**, 377 (1922).

[7] E. Kretschmann, "Über den physikalischen Sinn der Relativitätspostulate," *Ann. Phys.* **53**, 575 (1917).

[8] H. Bacry and J.-M. Lévy-Leblond, "Possible kinematics," *J. Math. Phys.* **9**, 1605 (1968).

[9] H. Bacry and J. Nuyts, "Classification of ten-dimensional kinematical groups with space isotropy," *J. Math. Phys.* **27**, 2455 (1986).

[10] E. Inönü and E. P. Wigner, "On the contraction of groups and their representations," *Proc. Natl. Acad. Sci.* **39**, 510 (1953).

[11] P. Breitenlohner and D. Z. Freedman, "Stability in gauged extended supergravity," *Ann. Phys.* **144**, 249 (1982).

[12] A. Ishibashi and R. M. Wald, "Dynamics in non-globally-hyperbolic static spacetimes: III. Anti-de Sitter spacetime," *Class. Quantum Grav.* **21**, 2981 (2004).

[13] A. G. Riess *et al.*, "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *Astron. J.* **116**, 1009 (1998).

[14] S. Perlmutter *et al.*, "Measurements of Ω and Λ from 42 high-redshift supernovae," *Astrophys. J.* **517**, 565 (1999).

[15] E. Mostaque, "One Postulate," Intelligent Internet (2026).